Proposed Research

We can't know what method of analysis will result in a proof (if we did, we'd be busy writing a proof). This is why, as we stated earlier, we will be employing a variety of methods. The following subsections delineate different methods of analysis which we plan to utilize, and the general result we hope to achieve from using them.

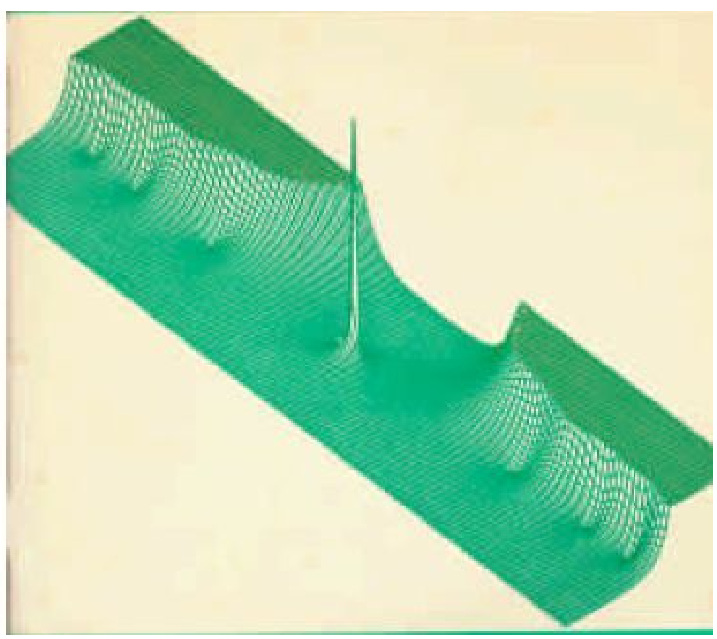
Complex Mappings of Zeta Function

In the background section we showed you the standard image that is associated with the Riemann zeta function. That graph is obtained by applying the zeta to the typical vertical and horizontal lines that make up a Cartesian grid. Linear transformations in the complex plane must be applied to a line, or many line, in order for people to see the transformation. So, the Riemann zeta function may be applied to any line in a plane, of which there are literally infinite. It doesn't stop at the complex plane. The function can be visualized in three dimensions as well, and technically four dimensions when you add the parameter of color. We intend to manipulate many different lines using the zeta function to see what kinds of fascinating transformations come about. The beauty of this analytical method is that there are endless possibilities and even an undergraduate student could conduct research this way. We will set our volunteer students free in this area to play around and see what they discover.

In her book, *Zeta Functions of Graphs, A Stroll Through the Garden*, Audrey Terras (UCSD) visualized the zeta function in many different ways. Here are two examples of what she found:

Figure 1

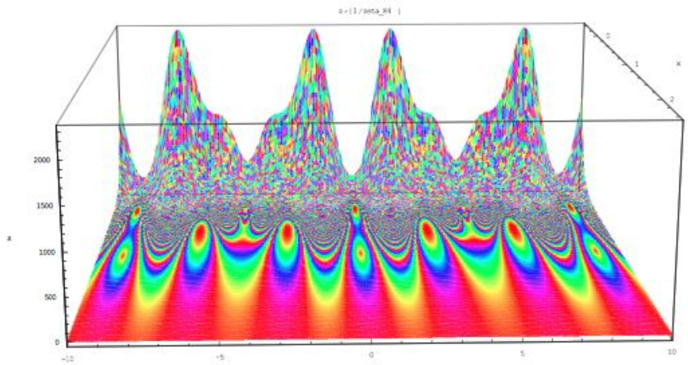
Graph of $z=|\zeta(x+iy)|$ showing the pole at $x+iy=1$ and the first 6 zeros which are on the line $x=1/2$, of course. The picture was made by D. Asimov and S. Wagon to accompany their article on the evidence for the Riemann hypothesis as of 1986.



Graph of $z=1/|\zeta_{K_4}(2^{-(x+iy)})|$

Drawn by Mathematica

Figure 2



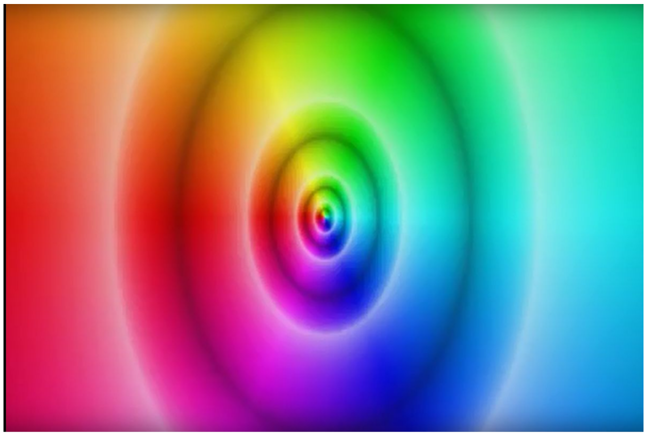
As you can see in figure 1, Terras maps the function into three dimensions, her z-axis values corresponding to the absolute value of the Riemann zeta function. This allows us to see the zeroes of the function more clearly, and the point at which the function is not defined, $x + iy = 1$, which is equivalent to x = 1 and y = 0.

Representing the function visually can make the problem easier to understand for people who are not as familiar with the hypothesis, but it can also offer a new perspective to mathematicians who've been studying this problem for years. Notice the stark differences in figure 2. The color comes from a graphing technique known as "domain coloring". The domain of the original complex plane is colored with a standard color wheel. Then after the function is applied, the colors are moved to their point's corresponding positions [13]. Notice that the physical shape of the graph is also different because Terras manipulated her z-axis in a different way. This time she set the values equal to the equation:

$$z = 1/\left|\zeta_{k_4}(2^{-(x+iy)})\right|$$

This particular transformation presents a new set of points with new trends to be analyzed. No matter how much math a person studies, they will not be able to visualize all the possible graphs of the zeta function, but these graphs can be obtained with current graphing technology. For these reasons, we intend to utilize graphing tools to visualize the zeta function. Once we do this, our minds will be the tool through which we try to make sense of what we see.

Let's take a look at exactly how we will be color mapping. The following example can be repeated using the "Wolfram Demonstrations Project" program. First we start with our complex plane with a color wheel placed at the center:



In this example, we will simply be mapping the function:

$$sin(x) \ = \ \sum_{n=1} \left((-1)^n \ * \ x^{(2n+1)\,/\,(2n+1)!} \right)$$

This is what our plane looks like after the sine function is applied:

At first this is a lot to take in, but notice the familiar sine wave that appears in white on the center horizontal line. This color map is unrelated to the Riemann Hypothesis, however it provides a simple example of a common method of research we will use in our work on the hypothesis. As previously stated, graphing the zeta function can reveal interesting patterns which will be far less obvious in the written equations. Wolfram is a tool that can be used by anybody with a computer, which makes it the perfect tool for our less experienced, volunteer students. The more the merrier when it comes to volunteers. It could not hurt to examine thousands of complex mappings of the Riemann zeta function; who knows when one will stand out. Thomas Edison once said, "When you have exhausted all possibilities, remember this - you haven't."